Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference
WFM03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The line $x = 8$ is a directrix of the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > 0, \ b > 0,$$

and the point (2, 0) is the corresponding focus.

Find the value of a and the value of b.

(5)

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Question 1 continued

Q1

(Total 5 marks)

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2. Use calculus to find the exact value of $\int_{-2}^{1} \frac{1}{x^2+4x+13}\,\mathrm{d}x$.

(5)

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Question 2 continued

Q2

(Total 5 marks)

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3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

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Question 3 continued

Q3

(Total 8 marks)

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4. $I_n = \int_0^a (a-x)^n \cos x \, \mathrm{d}x, \quad a > 0, \quad n \geqslant 0$

(a) Show that, for $n \geqslant 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(b) Hence evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, \mathrm{d}x$.

(3)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 8 marks)

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5. Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(a) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y$,

(5)

(b) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$.

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6. $\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}$, where k is a constant.

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$,

(a) find the eigenvalue of $\mathbf{M}$ corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$,

(2)

(b) show that $k = 3$,

(2)

(c) show that $\mathbf{M}$ has exactly two eigenvalues.

(4)

A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by $\mathbf{M}$.

The transformation T maps the line l_1, with cartesian equations $\frac{x-2}{1} = \frac{y}{-3} = \frac{z+1}{4}$, onto the line l_2.

(d) Taking $k = 3$, find cartesian equations of l_2.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 13 marks)

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7. The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(a) Find an equation of Π in the form $\mathbf{r.n} = p$, where $\mathbf{n}$ is a vector perpendicular to Π and p is a constant.

(5)

The point P has coordinates $(6, 13, 5)$. The line l passes through P and is perpendicular to Π. The line l intersects Π at the point N.

(b) Show that the coordinates of N are $(3, 1, -1)$.

(4)

The point R lies on Π and has coordinates $(1, 0, 2)$.

(c) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.

(5)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 14 marks)

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8. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

The line l_1 is the tangent to H at the point $P(4\sec t, 2\tan t)$.

(a) Use calculus to show that an equation of l_1 is

$$2y\sin t = x - 4\cos t$$

(5)

The line l_2 passes through the origin and is perpendicular to l_1.

The lines l_1 and l_2 intersect at the point Q.

(b) Show that, as t varies, an equation of the locus of Q is

$$\left(x^2 + y^2\right)^2 = 16x^2 - 4y^2$$

(8)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

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